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STOKES FLOWS INSIDE A SPHERE

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1. Pair of Stream Functions of Three-Dimensional Flow

Let Ω be a sphere of radius R with the center at the origin of coordinates, $\mathbf{e}_r$, $\mathbf{e}_\theta$, and $\mathbf{e}_\varphi$ be unit vectors of the spherical coordinate system (r, θ, φ), and $\mathbf{r} = r\mathbf{e}_r$. We designate the space of infinitely differentiable solenoidal vector fields in the closed sphere $\bar{\Omega}$ as V. In V we isolate the subspaces

$$V^- = \{\mathbf{v} \in V | \mathbf{v}\cdot\mathbf{e}_r = 0\},\ V^+ = \{\mathbf{v} \in V | \mathrm{rot}\,\mathbf{v}\cdot\mathbf{e}_r = 0\}.$$

With any function $F \in C^\infty(\bar{\Omega})$ one can associate a field $\mathbf{v}^-(F) = \mathrm{rot}\,F\mathbf{r} \in V^-$. Conversely, if a field $\mathbf{v}^- \in V^-$ is given, then from the condition div $\mathbf{v}^- = 0$ we get

$$\frac{\partial}{\partial\theta}\left(v_\theta^- \sin\theta\right) + \frac{\partial v_\varphi^-}{\partial\varphi} = 0,$$

and therefore there exists a function $F^- \in C^\infty(\bar{\Omega})$ such that

$$\frac{\partial F^-}{\partial\varphi} = v_\theta^- \sin\theta,\quad \frac{\partial F^-}{\partial\theta} = -v_\varphi^-.$$

It is verified that $\mathbf{v}^- = \mathbf{v}^-(F^-) = \mathrm{rot}\,F^-\mathbf{r}$.

With any function $F \in C^\infty(\bar{\Omega})$ one can also associate a field $\mathbf{v}^+(F) = \mathrm{rot}\,\mathrm{rot}\,F\mathbf{r} \in V^+$. If a field $\mathbf{v}^+ \in V^+$ is given, then from the condition rot $\mathbf{v}^+\cdot\mathbf{e}_r = 0$ we get

$$\frac{\partial}{\partial\theta}\left(v_\varphi^+ \sin\theta\right) - \frac{\partial v_\theta^+}{\partial\varphi} = 0,$$

and therefore a function $G \in C^\infty(\bar{\Omega})$ exists such that

$$\frac{\partial G}{\partial\varphi} = v_\varphi^+ \sin\theta,\quad \frac{\partial G}{\partial\theta} = v_\theta^+.$$

Defining $F^+(r, \theta, \varphi) = \frac{1}{r}\int_0^r \rho G(\rho, \theta, \varphi)\,d\rho$, we can verify that the angular field components rot rot $F^+\mathbf{r}$ coincide with the corresponding components $\mathbf{v}^+$, and since the radial component of the field $\mathbf{v}$ without a singularity at the origin of coordinates is uniquely expressed through the angular components from the condition div $\mathbf{v} = 0$, we have $\mathbf{v}^+ = \mathbf{v}^+(F^+) = \mathrm{rot}\,\mathrm{rot}\,F^+\mathbf{r}$.

The correspondences $F \to \mathbf{v}^-(F)$ and $F \to \mathbf{v}^+(F)$ defined above agree with the taking of the rot and lead to a scalar Laplace operator Δ = div grad: $C^\infty(\bar{\Omega}) \to C^\infty(\bar{\Omega})$ and a vector operator $\Delta = -$rot rot: $V \to V$. That is, the following equations are valid: rot $\mathbf{v}^-(F) = \mathbf{v}^+(F)$, rot $\mathbf{v}^+(F) = -\mathbf{v}^-(\Delta F)$, $\Delta\mathbf{v}^\pm(F) = \mathbf{v}^\pm(\Delta F)$. Since $\mathbf{v}^-(F) = \mathrm{rot}\,F\mathbf{r} = \mathrm{grad}\,F \times \mathbf{r}$, the streamlines of the field $\mathbf{v}^-(F)$ are intersections of surfaces F = const with spheres r = const; F is the stream function for $\mathbf{v}^-(F)$. If the field $\mathbf{v}^+(F)$ is irrotational, then $\frac{\partial}{\partial r}(rF)$ is its potential. If $\mathbf{v}^+(F)$ has axial symmetry, then its stream function has the form $\Psi = -r\sin\theta\,\partial F/\partial\theta$. Since all potential fields and all axisymmetric fields without twist and with the condition of solenoidality belong to V^+, F generalizes the stream function and the potential of the field $\mathbf{v}^+(F)$ at the same time.

We can show that any field $\mathbf{v} \in V$ is represented uniquely in the form $\mathbf{v} = \mathbf{v}^- + \mathbf{v}^+$, where $\mathbf{v}^- \in V^-$ and $\mathbf{v}^+ \in V^+$. We determine the function $F^+(r, \theta, \varphi)$ for each fixed r, $0 < r \le R$, as the solution of the equation $\Delta_{\theta\varphi}F = -rv_r$ satisfying the condition

$$\iint_{S_r} F^+ dS = 0, \tag{1.1}$$

where S_r is a sphere of radius r concentric with Ω and

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$$\Delta_{\theta,\varphi} = \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}$$

is the angular part of the Laplace operator. The condition of solvability of this equation is satisfied:

$$\iint_{S_r} r v_r dS = r\iint_{S_r} v_r dS = r\iiint_{\Omega_r} \operatorname{div} \mathbf{v} d\Omega = 0.$$

From the integral equation for the solution ([1], supplement II) we get $F^+ \in C^\infty(\bar\Omega)$. It is verified that the radial field component $\mathbf{v}^+ = \operatorname{rot}\operatorname{rot} F^+\mathbf{r} \in V^+$ coincides with $\mathbf{v}_r$, so that $\mathbf{v}^- = \mathbf{v} - \mathbf{v}^+ \in V^-$, and the sought expansion $\mathbf{v} = \mathbf{v}^- + \mathbf{v}^+$ has been obtained. To demonstrate its uniqueness it is sufficient to prove that $V^- \cap V^+ = \{0\}$. Let $\mathbf{v} \in V^- \cap V^+$. From $\mathbf{v} \in V^+$ it follows that $\mathbf{v} = \operatorname{rot}\operatorname{rot} F\mathbf{r}$, while from $\mathbf{v} \in V^-$, it follows that $v_r = -\frac{1}{r}\Delta_{\theta,\varphi}F = 0$, from which $F = F(r)$ and $\mathbf{v} = \operatorname{rot}\operatorname{rot} F\mathbf{r} = 0$. Thus, the following theorem has been proved.

<u>THEOREM 1.</u> For any vector field $\mathbf{v} \in V$ there exist two functions F^-, $F^+ \in C^\infty(\bar\Omega)$ such that

$$\mathbf{v} = \operatorname{rot} F^-\mathbf{r} + \operatorname{rot}\operatorname{rot} F^+\mathbf{r}.$$

The functions F^- and F^+ are determined with the accuracy of the addition of arbitrary functions of r. If we require that the condition (1.1) be satisfied for both functions $F^\pm$, then they are uniquely determined. For such functions $F^\pm$ the condition of impermeability of the boundary S of the sphere Ω has the form $F^+|_{r=R} = 0$, while the condition of attachment to S is $F^-|_{r=R} = F^+|_{r=R} = \left.\frac{\partial F^+}{\partial r}\right|_{r=R} = 0$.

The latter statements of the theorem are verified directly. The functions $F^\pm$ introduced allow one to find all the uniform helical flows inside the sphere, i.e., the solutions of the equation $\operatorname{rot}\mathbf{v} = \lambda\mathbf{v}$ with the condition of impermeability of the sphere S. For such a $\mathbf{v}$ the function F^- satisfies the Helmholtz equation $\Delta F^- = -\lambda^2 F^-$ and the boundary condition $F^-|_{r=R} = 0$, from which F^- is easily found [2]. The function F^+ is determined from the relation $F^- = \lambda F^+$. This course for seeking all the uniform helical flows inside a sphere is considerably shorter than the original one of [3].

2. Eigenvectors of the $\tilde\Delta$ Operator

The definition of the $\tilde\Delta$ operator and the Hilbert space $J^\circ(\Omega)$ in which it operates can be found in [4]. For our purposes it is essential that the eigenvectors of this operator be fields $\mathbf{v} \in V$, satisfying the system of equations

$$\tilde\Delta\mathbf{v} = \nu\Delta\mathbf{v} - \operatorname{grad} H = -\lambda\mathbf{v} \tag{2.1}$$

and the attachment condition

$$\mathbf{v}|_S = 0. \tag{2.2}$$

First let us find the eigenvectors $\mathbf{v} \in V^-$. Let $\mathbf{v} = \mathbf{v}^-(F) = \operatorname{rot} F\mathbf{r}$, where F satisfies the condition (1.1). Since $\Delta\mathbf{v}^-(F) = \mathbf{v}^-(\Delta F) \in V^-$, we have $\operatorname{grad} H = \nu\Delta\mathbf{v} + \lambda\mathbf{v} \in V^-$. On the other hand, $\operatorname{grad} H \in V^+$, and, from the proof in Sec. 1, grad H = 0. Therefore, (2.1) takes the form $\nu\Delta\mathbf{v} = -\lambda\mathbf{v}$. By virtue of the uniqueness of the reconstruction of F from $\mathbf{v}$ with the condition (1.1) and the identity $\Delta\mathbf{v}^-(F) = \mathbf{v}^-(\Delta F)$, we have

$$\nu\Delta F = -\lambda F. \tag{2.3}$$

The attachment condition (2.2) has the form

$$F|_{r=R} = 0. \tag{2.4}$$

It is well known [2] that the problem (2.3), (2.4) can have nontrivial solutions which are continuous at the origin of coordinates only when $\lambda = \lambda_{kn}^- = \nu\left(\frac{\mu_{kn}}{R}\right)^2$ and to each value λ_{kn}^- there correspond 2n+1 linearly independent solutions, for which we can take

$$F_{kn}^{m-} = \frac{C_{kn}^{m-}}{\sqrt{r}} J_{n+\frac{1}{2}}\left(\frac{\mu_{kn} r}{R}\right) Y_n^m(\theta,\varphi), \quad k, n \geqslant 1, \quad |m| \leqslant n \tag{2.5}$$

(the value n = 0 is excluded by virtue of (1.1)), where μ_{kn} is the k-th null of the Bessel function $J_{n+1/2}(z)$ in magnitude,

$$Y_n^m(\theta,\varphi) = P_n^m(\cos\theta)\cos m\varphi \quad \text{for} \quad 0 \leqslant m \leqslant n,$$

$$Y_n^m(\theta,\varphi) = P_n^{|m|}(\cos\theta)\sin|m|\varphi \quad \text{for} \quad -n \leqslant m < 0,$$

$$C_{kn}^{m-} = \frac{1}{R J_{n+\frac{3}{2}}(\mu_{kn})}\sqrt{\frac{1}{\pi\varepsilon_m}\frac{2n+1}{n(n+1)}\frac{(n-|m|)!}{(n+|m|)!}}, \quad \varepsilon_0 = 2, \ \varepsilon_m = 1 \ \text{for} \ m \neq 0.$$

The last constant is calculated from the normalization condition $\int\int\limits_{\Omega}\int v^2 d\Omega = 1$, where $v^2 = \mathbf{v}\cdot\mathbf{v}$ and $\mathbf{v} = \operatorname{rot} F_{kn}^{m-}\mathbf{r}$.

Now let us calculate the eigenvectors $v \in V^+$. Let $\mathbf{v} = \mathbf{v}^+(F) = \operatorname{rot}\operatorname{rot} F\mathbf{r}$, where F satisfies the condition (1.1). Applying the rot operator to both sides of (2.1), we obtain $\nu\Delta\omega = -\lambda\omega$, where $\omega = \operatorname{rot}\mathbf{v} = \operatorname{rot}\mathbf{v}^+(F) = -\mathbf{v}^-(\Delta F)$. Since, in addition to F, the function $G = \Delta F$ also satisfies the condition (1.1), by analogy with Eq. (2.3) we obtain

$$\nu\Delta G = -\lambda G. \tag{2.6}$$

Since $G \in C^\infty(\Omega)$, G is expanded into a uniformly converging series by spherical harmonics,

$$G = \sum_{n=1}^{\infty}\sum_{m=-n}^{n} g_n^m(r) Y_n^m(\theta, \varphi) = \sum_{n=1}^{\infty}\sum_{m=-n}^{n} G_n^m \tag{2.7}$$

(the term containing n = 0 is absent by virtue of (1.1)). The space of functions of the form $g(r)Y_n^m(\theta, \varphi)$ is invariant relative to the Laplace operator, so that each term of the series (2.7) is a solution of Eq. (2.6), and hence equals

$$G_n^m = \frac{C_n^m}{\sqrt{r}} J_{n+\frac{1}{2}}\left(\sqrt{\frac{\lambda}{\nu}}\, r\right) Y_n^m(\theta, \varphi).$$

Let $F = \sum_{n=1}^{\infty}\sum_{m=-n}^{n} F_n^m$ be an expansion analogous to (2.7). It follows from Eq. (2.6) that the function $H_n^m = \frac{\lambda}{\nu} F_n^m +$ G_n^m is harmonic. The attachment condition (2.2) has the form

$$F|_{r=R} = \partial F/\partial r|_{r=R} = 0. \tag{2.8}$$

From the uniqueness of the expansions of F and $\partial F/\partial r$ by spherical harmonics at r = R it follows that the condition (2.8) is also valid for each F_n^m. We have

$$H_n^m|_{r=R} = G_n^m|_{r=R} = \frac{C_n^m}{\sqrt{R}} J_{n+\frac{1}{2}}\left(\sqrt{\frac{\lambda}{\nu}}\, R\right) Y_n^m(\theta, \varphi).$$

A harmonic function H_n^m satisfying this condition can be chosen in the form

$$H_n^m = \frac{C_n^m}{\sqrt{R}} J_{n+\frac{1}{2}}\left(\sqrt{\frac{\lambda}{\nu}}\, R\right)\left(\frac{r}{R}\right)^n Y_n^m(\theta, \varphi).$$

This function is uniquely determined by virtue of the uniqueness of the solution of the Dirichlet problem for the Laplace equation. The condition $\left.\frac{\partial F_n^m}{\partial r}\right|_{r=R} = 0$ gives $J_{n+\frac{3}{2}}\left(\sqrt{\frac{\lambda}{\nu}}\, R\right) = 0$, from which $\lambda = \lambda_{kn}^+ = \nu\left(\frac{\mu_{k,n+1}}{R}\right)^2$,

$$F_{kn}^{m+} = \frac{\nu}{\lambda}(H_n^m - G_n^m) = \left(\frac{R}{\mu_{k,n+1}}\right)^2 C_{kn}^{m+}\left[\frac{1}{\sqrt{R}} J_{n+\frac{1}{2}}(\mu_{k,n+1})\left(\frac{r}{R}\right)^n - \frac{1}{\sqrt{r}} J_{n+\frac{1}{2}}\left(\frac{\mu_{k,n+1} r}{R}\right)\right] Y_n^m(\theta, \varphi). \tag{2.9}$$

Since the functions $J_{n+3/2}(z)$ do not have common nulls for different n, only 2n+1 terms corresponding to a fixed n can take part in the expansion (2.7) and the analogous expansion for F. Thus, to each value of λ_{kn}^+ there correspond 2n+1 linearly independent solutions of the problem (2.1), (2.2), for which one can take the fields $\mathbf{v} = \operatorname{rot}\operatorname{rot} F_{kn}^{m+}\mathbf{r}$, where F_{kn}^{m+} is given by Eq. (2.9) with $k, n \geq 1$, $|m| \leq n$, and

$$C_{kn}^{m+} = \frac{\mu_{k,n+1}}{R^2 J_{n+\frac{5}{2}}(\mu_{k,n+1})}\sqrt{\frac{1}{\pi\varepsilon_m}\,\frac{2n+1}{n(n+1)}\,\frac{(n-|m|)!}{(n+|m|)!}}.$$

Now let us consider an arbitrary eigenvector $v \in V$. We have $\mathbf{v} = \mathbf{v}^- + \mathbf{v}^+$, and, by virtue of the invariance of the subspaces V^- and V^+ relative to the $\tilde{\Delta}$ operator, the fields $\mathbf{v}^-$ and $\mathbf{v}^+$ themselves are eigenvectors with one and the same eigenvalues. For n > 1 this value is $\lambda_{kn}^- = \nu\left(\frac{\mu_{kn}}{R}\right)^2 = \lambda_{k,n-1}^+$. In this case for each $k \geq 1$ $\mathbf{v}$ is a linear combination of 4n fields, already constructed, of the form $\operatorname{rot} F_{kn}^{m-}\mathbf{r}$, $|m| \leq n$, and $\operatorname{rot}\operatorname{rot} F_{k,n-1}^{m+}\mathbf{r}$, $|m| \leq n-1$. For n = 1 and an arbitrary k we have $\mathbf{v}^+ = 0$, while $\mathbf{v}^-$ is a linear combination of three fields of the form $\operatorname{rot} F_{k1}^{m-}\mathbf{r}$, m = −1, 0, 1. We also note that the spaces V^- and V^+ are orthogonal in the sense of the scalar product

$$\langle \mathbf{V}, \mathbf{W} \rangle = \iiint_{\Omega} \mathbf{V} \cdot \mathbf{W} d\Omega.$$

Using the properties of the $\tilde{\Delta}$ operator demonstrated in [4], we obtain the following result.

THEOREM 2. Fields of the form $\mathbf{v}_{kn}^{m-} = \operatorname{rot} F_{kn}^{m-}\mathbf{r}$ and $\mathbf{v}_{kn}^{m+} = \operatorname{rot}\operatorname{rot} F_{kn}^{m+}\mathbf{r}$, where $k, n \geq 1$, $|m| \leq n$, while the functions $F_{kn}^{m\pm}$ are assigned by Eqs. (2.5) and (2.9), respectively, form an orthonormal basis in the space $J^{\circ}(\Omega)$.

3. Solution of the Stokes System of Equations

First let us consider the case of potential mass forces $\mathbf{F} = -\operatorname{grad} U$. The Stokes equations have the vector form

$$\partial \mathbf{v}/\partial t = \nu \Delta \mathbf{v} - \operatorname{grad} H, \tag{3.1}$$

where $H = p/\rho + v^2/2 + U$ is Lamb's function. To the system (3.1) we must add the continuity equation

$$\operatorname{div} \mathbf{v} = 0, \tag{3.2}$$

the attachment condition

$$\mathbf{v}|_S = 0, \tag{3.3}$$

and the initial condition

$$\mathbf{v}|_{t=0} = \mathbf{v}_0. \tag{3.4}$$

The problem (3.1)-(3.4) can be treated as the problem of finding the field $\mathbf{v}$ belonging to the region of definition of the $\tilde{\Delta}$ operator satisfying the equation

$$\partial \mathbf{v}/\partial t = \tilde{\Delta} \mathbf{v} \tag{3.5}$$

and the initial condition (3.4) (see [4]). The solution of the latter problem is represented by the Fourier series

$$\mathbf{v}(t) = \sum_{k,n=1}^{\infty} \sum_{m=-n}^{n} \left[a_{kn}^m \exp\left(-\lambda_{kn}^- t\right) \mathbf{v}_{kn}^{m-} + b_{kn}^m \exp\left(-\lambda_{kn}^+ t\right) \mathbf{v}_{kn}^{m+}\right], \tag{3.6}$$

where $a_{kn}^m = \langle \mathbf{v}_0, \mathbf{v}_{kn}^{m-} \rangle = \iiint_{\Omega} \mathbf{v}_0 \cdot \mathbf{v}_{kn}^{m-} d\Omega$; $b_{kn}^m = \langle \mathbf{v}_0, \mathbf{v}_{kn}^{m+} \rangle = \iiint_{\Omega} \mathbf{v}_0 \cdot \mathbf{v}_{kn}^{m+} d\Omega$. It was shown in [4] that the series (3.6) satisfies Eqs. (3.1) and (3.2) and the boundary condition (3.3) with $t > 0$ for any $\mathbf{v}_0 \in J^{\circ}(\Omega)$, with the initial condition (3.4) being satisfied in the sense of $\mathbf{v}(t)$ tending toward $\mathbf{v}_0$ as $t \to +0$ with respect to the norm of the space $J^{\circ}(\Omega)$. In order for the series (3.6) to represent the classical solution of the problem (3.1)-(3.4) it is sufficient that the field $\mathbf{v}_0$ be twice continuously differentiable up to the boundary of Ω and satisfy the conditions (3.2) and (3.3).

In the case of an arbitrary mass force field, depending on time, generally speaking, under the assumption of single continuous differentiability of this field inside $\overline{\Omega}$ we represent it in the form $\mathbf{F} = \mathbf{F}_1 - \operatorname{grad} U$, where U is the solution of the Neumann problem

$$\Delta U = -\operatorname{div} \mathbf{F}, \tag{3.7}$$

$$\left.\frac{\partial U}{\partial n}\right|_S = -\mathbf{F} \cdot \mathbf{n}, \tag{3.8}$$

and $\mathbf{n}$ is the unit vector of the outer normal to S. Then to the right sides of Eqs. (3.1) and (3.5) we add the term $\mathbf{F}_1$, which satisfies the conditions $\operatorname{div} \mathbf{F}_1 = 0$ and $\mathbf{F}_1 \cdot \mathbf{n} = 0$ by virtue of (3.7) and (3.8) and therefore belongs to $J^{\circ}(\Omega)$. Solving the equations obtained together with the conditions (3.2)-(3.4), just like the boundary problem for the inhomogeneous equation of heat conduction in [1] or [2], we find that to the series (3.6) we must add the series

$$\sum_{k,n=1}^{\infty} \sum_{m=-n}^{n} \left\{\left(\int_0^t \exp\left[-\lambda_{kn}^- (t-\tau)\right] f_{kn}^m(\tau)\, d\tau\right) \mathbf{v}_{kn}^{m-} + \left(\int_0^t \exp\left[-\lambda_{kn}^+ (t-\tau)\right] g_{kn}^m(\tau)\, d\tau\right) \mathbf{v}_{kn}^{m+}\right\},$$

where $f_{kn}^m(t) = \langle \mathbf{F}_1, \mathbf{v}_{kn}^{m-} \rangle = \iiint_{\Omega} \mathbf{F}_1 \cdot \mathbf{v}_{kn}^{m-} d\Omega$;

$$g_{kn}^m(t) = \langle \mathbf{F}_1, \mathbf{v}_{kn}^{m+} \rangle = \iiint_{\Omega} \mathbf{F}_1 \cdot \mathbf{v}_{kn}^{m+} d\Omega.$$

From the expansion (3.6) it is seen that in the case of potential mass forces, some time after the start of the flow, regardless of the initial condition, the lowest harmonic will predominate in it,

$$\sum_{m=-1}^{1} a_{11}^{m} \exp\left(-\lambda_{11}^{-} t\right) \mathbf{v}_{11}^{m-};$$

it represents differential rotation about some axis proportional to the aximuthal component of a helical Hill vortex, described in [5], and damped in proportion to exp $(-20.19\nu R^{-2}t)$.

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NONISOTHERMAL FLOW INDUCED BY THE SQUEEZING OF A NON-NEWTONIAN FLUID FILM BETWEEN TWO PARALLEL PLATES

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In the pressure forming of thin-sheet products, the polymer melt is injected into the cavity formed by partially contacting half-molds. The next stage is the joining of the half-molds, during which time the melt is squeezed to fill the mold cavity and harden at the end of the process.

Here we consider the problem of nonisothermal flow induced in a molten polymer film between two parallel plates (half-molds), which are squeezed together at a rate v in the direction normal to the plane of the plates. We investigate the temperature regime of the fluid cooling process as a function of the governing parameters of the problem.

It is assumed that the fluid is incompressible and obeys a power rheological law, where the consistency depends on the temperature T: $\mu = \mu(T)$.

The temperature of the fluid at the initial time is T_0, and the wall temperature is T_W $(T_W \ll T_0)$.

To the best of our knowledge, this kind of problem has been investigated only in [1]. However, to simplify the solution the authors have, without justification, rejected the convection term in the heat-balance equation.

We introduce a cylindrical coordinate system with the z axis directed perpendicular to the plane of the plates and with the origin situated at the center of the lower plate (Fig. 1). The radius of the fluid film R(t) is a function of the time t and is determined from the condition of a constant initial volume of the fluid.

Taking axial symmetry into account, we find that the tangential component of the velocity v_φ and the derivatives of all variables with respect to φ are equal to zero.

Under the condition that body forces and surface-tension forces are negligible, the stated problem corresponds to the system of equations

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